FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Wednesday 24th February, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. For the statements p and q, consider the following compound statements :

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct?

(1) (a) and (b) both are not tautologies.

(2) (a) and (b) both are tautologies.

(3) (a) is a tautology but not (b).

(4) (b) is a tautology but not (a).

Official Ans. by NTA (2)

Sol. (A)

p	q	~ q	p → q	~ p	(~ q ∧ (p → q))	
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T

(B)

p	q	p ∨ q	~ p	(p ∨ q) ∧ ~ p	
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Both are tautologies

2. Let a, b ∈ R. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is (20, b, –a –9), then |a + b| is equal to :

(1) 88 (2) 86

(3) 84 (4) 90

Official Ans. by NTA (1)

Sol. P(9, 6, 9)

$$\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$$

Q = (20, b, –a – 9)

$$\frac{\frac{20+a}{2}-3}{7}=\frac{\frac{b+6}{2}-2}{5}=\frac{-\frac{9}{2}-1}{-9}$$

$$\frac{14+9}{14}=\frac{b+2}{10}=\frac{a+2}{18}$$

⇒ a = –56 and b = –32

⇒ |a + b| = 88

3. The vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}.(\hat{i}-2\hat{j})=-2$, and the point (1, 0, 2) is :

(1) $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=\frac{7}{3}$

(2) $\vec{r}.(3\hat{i}+7\hat{j}+3\hat{k})=7$

(3) $\vec{r}.(\hat{i}+7\hat{j}+3\hat{k})=7$

(4) $\vec{r}.(\hat{i}-7\hat{j}+3\hat{k})=\frac{7}{3}$

Official Ans. by NTA (3)

Sol. $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$

$\vec{r}.(\hat{i}-2\hat{j})=-2$

point (1, 0, 2)

Eqn of plane

$$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)-1+\lambda\left\{r.(\hat{i}-2\hat{j})+2\right\}=0$$

$$\vec{r}.\left\{\hat{i}(1+\lambda)+\hat{j}(1-2\lambda)+\hat{k}(1)\right\}-1+2\lambda=0$$

Point $\hat{i}+0\hat{j}+2\hat{k}=\vec{r}$

$$\therefore\ (\hat{i}+2\hat{k}).\left\{\hat{i}(1+\lambda)+\hat{j}(1-2\lambda)+\hat{k}(1)\right\}-1+2\lambda=0$$

$1+\lambda+2-1+2\lambda=0$

$\lambda=-\frac{2}{3}$

$$\therefore\ \vec{r}.\left[\hat{i}\left(\frac{1}{3}\right)+\hat{j}\left(\frac{7}{3}\right)+\hat{k}\right]=\frac{7}{3}$$

$$r.\left[\hat{i}+7\hat{j}+3\hat{k}\right]=7$$

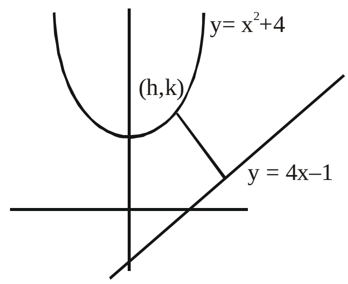
Ans. 3

4. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :

(1) (3, 13) (2) (1, 5)
(3) (–2, 8) (4) (2, 8)

Official Ans. by NTA (4)

Sol. Ans. (4)



P : $y = x^2 + 4$
$k = h^2 + 4$
L : $y = 4x - 1$
$y - 4x + 1 = 0$

$$d = AB = \left|\frac{k-4h+1}{\sqrt{5}}\right| = \left|\frac{h^2-4-4h+1}{\sqrt{5}}\right|$$

$$\frac{d(d)}{dh} = \frac{2h-4}{\sqrt{5}} = 0$$

$h = 2$

$$\frac{d^2(d)}{dh^2} = \frac{2}{\sqrt{5}} > 0$$

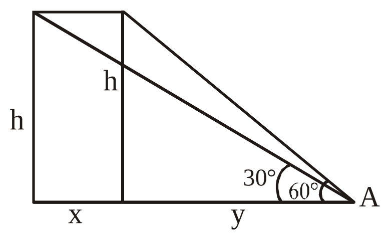
$\therefore$ $k = 4 + 4 = 8$
$\therefore$ Point (2, 8)

5. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is :

(1) $1800\sqrt{3}$ m (2) $3600\sqrt{3}$ m
(3) $2400\sqrt{3}$ m (4) $1200\sqrt{3}$ m

Official Ans. by NTA (4)

Sol.



$$\tan 60° = \frac{h}{y}$$

$$\sqrt{3} = \frac{h}{y} \Rightarrow h = \sqrt{3}y \quad .....(1)$$

$$\tan 30° = \frac{h}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+y} \Rightarrow \sqrt{3}h = x + y \quad ......(2)$$

Speed 432 km/h $\Rightarrow \frac{432 \times 20}{60 \times 60} \Rightarrow \frac{12}{5}$km

$$\sqrt{3}h = \frac{12}{5} + y$$

$$\sqrt{3}h - \frac{12}{5} = y$$

from (1)

$$h = \sqrt{3}\left[\sqrt{3}h - \frac{12}{5}\right]$$

$$h = 3h - \frac{12\sqrt{3}}{5}$$

$$h = \frac{6\sqrt{3}}{5}\text{km}$$

$$h = 1200\sqrt{3}\,\text{m}$$

6. If $n \ge 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2)$ is:

(1) $\frac{n(n-1)(2n+1)}{6}$ (2) $\frac{n(n+1)(2n+1)}{6}$

(3) $\frac{n(2n+1)(3n+1)}{6}$ (4) $\frac{n(n+1)^2(n+2)}{12}$

Official Ans. by NTA (2)

Sol. ${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2\right)$

${}^{n+1}C_2 + 2\left({}^3C_3 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2\right)$

$\left\{\text{use } {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_r\right\}$

$= {}^{n+1}C_2 + 2\left({}^4C_3 + {}^4C_2 + {}^5C_3 + ... + {}^nC_2\right)$

$= {}^{n+1}C_2 + 2\left({}^5C_3 + {}^5C_2 + ... + {}^nC_2\right)$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$= {}^{n+1}C_2 + 2\left({}^nC_3 + {}^nC_2\right)$

$= {}^{n+1}C_2 + 2.{}^{n+1}C_3$

$= \dfrac{(n+1)n}{2} + 2.\dfrac{(n+1)(n)(n-1)}{2.3}$

$= \dfrac{n(n+1)(2n+1)}{6}$

7. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be defined as,

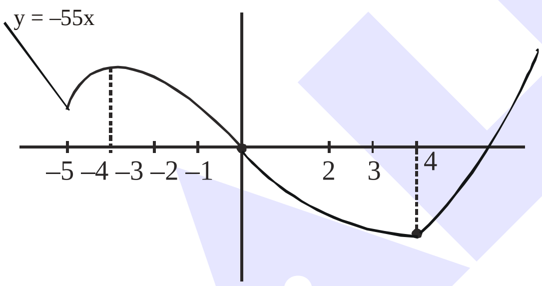
$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let $A = \{x \in \mathbf{R} : f \text{ is increasing}\}$. Then A is equal to :

(1) $(-\infty, -5) \cup (4, \infty)$
(2) $(-5, \infty)$
(3) $(-\infty, -5) \cup (-4, \infty)$
(4) $(-5, -4) \cup (4, \infty)$

Official Ans. by NTA (4)

Sol.



$$f'(x) = \begin{cases} -55\,; & x < -5 \\ 6(x-5)(x+4)\,; & -5 < x < 4 \\ 6(x-3)(x+2)\,; & x > 4 \end{cases}$$

f(x) is increasing in

$x \in (-5, -4) \cup (4, \infty)$

8. Let f be a twice differentiable function defined on R such that $f(0) = 1$, $f'(0) = 2$ and $f'(x) \neq 0$ for all $x \in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in R$, then the value of f(1) lies in the interval:

(1) (9, 12) (2) (6, 9)
(3) (0, 3) (4) (3,6)

Official Ans. by NTA (2)

Sol. $f(x)f''(x) - (f'(x))^2 = 0$

$\dfrac{f''(x)}{f'(x)} = \dfrac{f'(x)}{f(x)}$

$\ln(f'(x)) = \ln f(x) + \ln c$

$f'(x) = cf(x)$

$\dfrac{f'(x)}{f(x)} = c$

$\ln f(x) = cx + k_1$

$f(x) = ke^{cx}$

$f(0) = 1 = k$

$f'(0) = c = 2$

$f(x) = e^{2x}$

$f(1) = e^2 \in (6, 9)$

9. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\dfrac{3\sqrt{3}}{2}, \dfrac{1}{2}\right)$?

(1) $x^2 + y^2 = 7$ (2) $y^2 = \dfrac{1}{6\sqrt{3}}x$

(3) $2x^2 - 18y^2 = 9$ (4) $x^2 + 9y^2 = 9$

Official Ans. by NTA (4)

Sol. $m = -\dfrac{1}{\sqrt{3}}, c = 2$

(1) $c = a\sqrt{1+m^2}$

$c = \sqrt{7}\dfrac{2}{\sqrt{3}}$ (incorrect)

(2) $c = \dfrac{a}{m} = \dfrac{\dfrac{1}{24\sqrt{3}}}{\dfrac{-1}{\sqrt{3}}} = -\dfrac{1}{24}$ (incorrect)

(3) $c = \sqrt{a^2m^2 - b^2}$

$c = \sqrt{\dfrac{9}{2}\cdot\dfrac{1}{3} - \dfrac{1}{2}} = 1$ (incorrect)

(4) $c = \sqrt{a^2m^2 + b^2}$

$c = \sqrt{9\cdot\dfrac{1}{3} + 1} = 2$ (correct)

10. The value of the integral, $\int_1^3 [x^2-2x-2]\,dx$, where [x] denotes the greatest integer less than or equal to x, is :

(1) $-\sqrt{2}-\sqrt{3}+1$ (2) $-\sqrt{2}-\sqrt{3}-1$
(3) –5 (4) –4

Official Ans. by NTA (2)

Sol. $\int_1^3\left(\left[(x-1)^2\right]-3\right)dx$

$=\int_1^2[x^2]-3\int_1^1 dx$

$=\int_1^3 0\,dx+\int_1^{\sqrt{2}}1.dx+\int_{\sqrt{2}}^{\sqrt{3}}2.dx+\int_{\sqrt{3}}^{2}3.dx-6$

$=\sqrt{2}-1+2(\sqrt{3}-\sqrt{2})+3(2-\sqrt{3})-6$

$=-\sqrt{2}-\sqrt{3}-1$

11. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is :

(1) $\frac{1}{\sqrt{7}}$ (2) $2\sqrt{2}-1$

(3) $\sqrt{7}-1$ (4) $\frac{1}{2\sqrt{2}}$

Official Ans. by NTA (1)

Sol. Let $\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}=\theta$

$\sin 4\theta=\frac{\sqrt{63}}{8}$

$\cos 4\theta=\frac{1}{8}$

$2\cos^2 2\theta-1=\frac{1}{8}$

$\cos^2 2\theta=\frac{9}{16}$

$\cos 2\theta=\frac{3}{4}$

$2\cos^2\theta-1=\frac{3}{4}$

$\cos^2\theta=\frac{7}{8}$

$\cos\theta=\frac{\sqrt{7}}{2\sqrt{2}}$

$\tan\theta=\frac{1}{\sqrt{7}}$

12. The negative of the statement $\sim p \wedge (p \vee q)$ is

(1) $\sim p \vee q$ (2) $p \vee \sim q$
(3) $\sim p \wedge q$ (4) $p \wedge \sim q$

Official Ans. by NTA (2)

Sol. $\sim(\sim p \wedge (p \vee q))$

$p \vee (\sim p \wedge \sim q)$

$\underbrace{(p \vee \sim p)}_{t} \wedge (p \vee \sim q)$

$p \vee \sim q$

13. If the curve $y = ax^2 + bx + c$, $x \in R$, passes through the point (1,2) and the tangent line to this curve at origin is y = x, then the possible values of a, b, c are :

(1) $a=\frac{1}{2}$, $b=\frac{1}{2}$, c = 1

(2) a = 1, b = 0, c= 1
(3) a = 1, b = 1, c = 0
(4) a = –1, b = 1, c = 1

Official Ans. by NTA (3)

Sol. a + b + c = 2 ...(1)

and $\left.\frac{dy}{dx}\right|_{(0,0)}=1$

$2ax+b\big|_{(0,0)}=1$

b = 1
Curve passes through origin
So, c = 0
and a = 1

14. The area of the region :
$R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is :

(1) $11\sqrt{3}$ square units

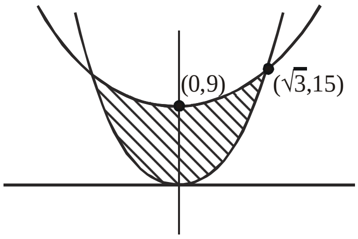
(2) $12\sqrt{3}$ square units

(3) $9\sqrt{3}$ square units

(4) $6\sqrt{3}$ square units

Official Ans. by NTA (2)

Sol.



Required area $= 2\int_0^{\sqrt{3}} (2x^2+9-5x^2)\,dx$

$= 2\left[9x - x^3\right]_0^{\sqrt{3}}$

$= 2\left[9\sqrt{3} - 3\sqrt{3}\right] = 12\sqrt{3}$

15. If a curve y = f(x) passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$?

(1) 5 (2) 10

(3) $\frac{62}{5}$ (4) $\frac{31}{5}$

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$

I.F. $= e^{\int \frac{1}{x}dx} = x$

So, solution of D.E. is given by

$y.x = \int b.x^3.x\,dx + c$

$y = \frac{c}{x} + \frac{bx^4}{5}$

Passes through (1, 2)

$2 = c + \frac{b}{5}$ ...(1)

$\int_1^2 f(x)dx = \frac{62}{5}$

$\left[c\ln x + \frac{bx^5}{25}\right]_1^2 = \frac{62}{5}$

$c\ln 2 + \frac{31b}{25} = \frac{62}{5}$ ...(2)

By equation (1) & (2)

c = 0 and b = 10

16. Let f(x) be a differentiable function defined on [0, 2] such that f '(x) = f '(2 – x) for all x ∈ (0, 2), f(0) = 1 and f(2) = e^2. Then the value of $\int_0^2 f(x)\,dx$ is :

(1) $1 - e^2$ (2) $1 + e^2$

(3) $2(1 - e^2)$ (4) $2(1 + e^2)$

Official Ans. by NTA (2)

Sol. $f'(x) = f'(2-x)$

f(x) = – f(2 – x) + c

put x = 0

$f'(0) = -f'(2) + c$

$c = f(0) + f(2) = 1 + e^2$

so, $f(x) + f(2 - x) = 1 + e^2$

$I = \int_0^2 f(x)\,dx$

$I = \int_0^2 f(2-x)\,dx$

$2I = \int_0^2 \left(f(x) + f(2-x)\right)dx$

$2I = (1+e^2)\int_0^2 dx$

$I = 1 + e^2$

17. Let A and B be 3 × 3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3 × 1 column matrix of unknown variables and O is a 3 × 1 null matrix, has :

(1) no solution

(2) exactly two solutions

(3) infinitely many solutions

(4) a unique solution

Official Ans. by NTA (3)

Sol. Let $A^T = A$ and $B^T = -B$

$C = A^2B^2 - B^2A^2$

$C^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T(A^2)^T - (A^2)^T(B^2)^T$

$= B^2A^2 - A^2B^2$

$C^T = -C$

C is skew symmetric.

So det(C) = 0

so system have infinite solutions.

18. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is :

(1) $\frac{71}{256}$ (2) $\frac{69}{256}$

(3) $-\frac{69}{256}$ (4) $-\frac{71}{256}$

Official Ans. by NTA (4)

Sol. $\frac{a+2+a}{3} = \frac{10}{3}$

a = 4

and $\frac{c+b+b}{3} = \frac{7}{3}$

c + 2b = 7

also 2b = a + c

2b – a + 2b = 7

$b = \frac{11}{4}$

now $4x^2 + \frac{11}{4}x + 1 = 0 \;\; \genfrac{}{}{0pt}{}{\alpha}{\beta}$

$\alpha^2 + \beta^2 - \alpha\beta = (\alpha + \beta)^2 - 3\alpha\beta$

$= \left(\frac{-11}{16}\right)^2 - 3\left(\frac{1}{4}\right)$

$= \frac{121}{256} - \frac{3}{4} = \frac{-71}{256}$

19. For the system of linear equations :

$x - 2y = 1$, $x - y + kz = -2$, $ky + 4z = 6$, $k \in R$,

consider the following statements :

(A) The system has unique solution if $k \neq 2$, $k \neq -2$.

(B) The system has unique solution if $k = -2$.

(C) The system has unique solution if $k = 2$.

(D) The system has no-solution if $k = 2$.

(E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct ?

(1) (C) and (D) only

(2) (B) and (E) only

(3) (A) and (E) only

(4) (A) and (D) only

Official Ans. by NTA (4)

Sol. $D = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$

so, A is correct and B, C, E are incorrect.

If k = 2

$D_1 = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = -48 \neq 0$

So no solution

D is correct.

20. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is :

(1) $\frac{65}{2^7}$ (2) $\frac{65}{2^8}$

(3) $\frac{135}{2^9}$ (4) $\frac{35}{2^7}$

Official Ans. by NTA (3)

Sol. Total subsets = 2^5 = 32

Probability = $\frac{{}^5C_2 \times 3^3}{32 \times 32} = \frac{10 \times 27}{12^{10}} = \frac{135}{2^9}$

SECTION-B

1. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to ________.

Official Ans. by NTA (12)

Ans. by ALLEN (BONUS)

Sol. **Bonus**

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$

${}^{25}C_k + {}^{25}C_{k+1}$

${}^{26}C_{k+1}$

as ${}^nC_r$ is defined for all values of n as will as r so ${}^{26}C_{k+1}$ always exists

Now k is unbounded so maximum value is not defined.

2. Let λ be an interger. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is ________.

Official Ans. by NTA (1)

Sol. $\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z-0}{-\frac{1}{2}}$

$\frac{x-0}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$

Shortest distance = $\frac{(a_2 - a_1).(b_1 \times b_2)}{|b_1 \times b_2|}$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(\frac{1}{2}+\frac{1}{2}\right) - \hat{j}\left(1+\frac{1}{2}\right) + \hat{k}\left(1-\frac{1}{2}\right)$$

$$= \hat{i} - \frac{3}{2}\hat{j} + \frac{\hat{k}}{2} = \frac{2\hat{i}-3\hat{j}+\hat{k}}{2}$$

$$\frac{b_1 \times b_2}{|b_1 \times b_2|} = \frac{2\hat{i}-3\hat{j}+\hat{k}}{\sqrt{14}}$$

$$\frac{(a_2-a_1).(b_1 \times b_2)}{|b_1 \times b_2|} = \left(-\lambda\hat{i} + \left(-2\lambda + \frac{1}{2}\right) + \lambda\hat{k}\right)\left(\frac{2\hat{i}-3\hat{j}+\hat{k}}{\sqrt{14}}\right)$$

$$= \left|\frac{-2\lambda + 6\lambda - \frac{3}{2} + \lambda}{\sqrt{14}}\right| = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\left|5\lambda - \frac{3}{2}\right| = \frac{7}{2}$$

$$5\lambda = \frac{3}{2} \pm \frac{7}{2}$$

$5\lambda = 5, -2$

$\lambda = 1, -\frac{2}{5}$

3. If $a + \alpha = 1$, $b + \beta = 2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$, $x \neq 0$, then the value of expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is ________.

Official Ans. by NTA (2)

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ ....(1)

replace x by $\frac{1}{x}$

$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$ ....(2)

(1) + (2)

$$(a+\alpha)f(x) + (a+\alpha)f\left(\frac{1}{x}\right) = x(b+\beta) + (b+\beta)\frac{1}{x}$$

$$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}} = \frac{b+\beta}{a+\alpha} = \frac{2}{1} = 2$$

4. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (– 5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to ________.

Official Ans. by NTA (56)

Sol. Let point is (h, k)

So, $\sqrt{(h-5)^2 + k^2} = 3\sqrt{(h+5)^2 + k^2}$

$8x^2 + 8y^2 + 100x + 200 = 0$

$x^2 + y^2 + \frac{25}{2}x + 25 = 0$

$$r^2 = \frac{(25)^2}{4^2} - 25$$

$$4r^2 = \frac{25^2}{4} - 100$$

$4r^2 = 156.25 - 100$

$4r^2 = 56.25$

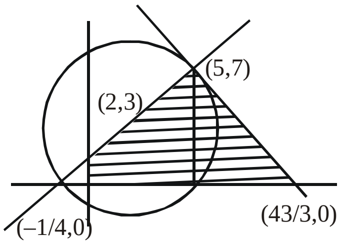
After round of $4r^2 = 56$

5. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2 + (y-3)^2 = 25$ at the point (5, 7) is A, then 24A is equal to_____.

Official Ans. by NTA (1225)

Ans. by ALLEN (1225 / BONUS)

Sol.



Equation of normal

$4x - 3y + 1 = 0$

and equation of tangents

$3x + 4y - 43 = 0$

Area of triangle $= \frac{1}{2}\left(\frac{43}{3}+\frac{1}{4}\right)\times(7)$

$= \frac{1}{2}\left(\frac{172+3}{12}\right)\times 7$

$A = \frac{1225}{24}$

$24A = 1225$

* as positive x-axis is given in the question so question should be bonus.

6. If the variance of 10 natural numbers 1, 1, 1,...., 1, k is less than 10, then the maximum possible value of k is ______.

Official Ans. by NTA (11)

Sol. $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$= \frac{9+k^2}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

$90 + 10k^2 - 81 - k^2 - 18k < 1000$

$9k^2 - 18k - 991 < 0$

$k^2 - 2k < \frac{991}{9}$

$(k-1)^2 < \frac{1000}{9}$

$\frac{-10\sqrt{10}}{3} < k - 1 < \frac{10\sqrt{10}}{3}$

$k < \frac{10\sqrt{10}}{3} + 1$

$k \le 11$

Maximum value of k is 11.

7. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is _______.

Official Ans. by NTA (3)

Sol. Let number are a, ar, ar^2, ar^3

$a\frac{(r^4-1)}{r-1} = \frac{65}{12}$ ...(1)

$\frac{1}{a}\frac{\left(\frac{1}{r^4}-1\right)}{\frac{1}{r}-1} = \frac{65}{18}$

$\frac{1}{ar^3}\left(\frac{1-r^3}{1-r}\right) = \frac{65}{18}$ ....(2)

$\frac{(1)}{(2)} \Rightarrow a^2r^3 = \frac{3}{2}$

and $a^3.r^3 = 1$

$ar = 1$

$(ar)^2.r = \frac{3}{2}$

$r = \frac{3}{2}$, $a = \frac{2}{3}$

So, third term $= ar^2 = \frac{2}{3}\times\frac{9}{4}$

$\alpha = \frac{3}{2}$

$2\alpha = 3$

8. The students $S_1, S_2, \ldots\ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _____.

Official Ans. by NTA (31650)

Sol. If group C has one student then number of groups

${}^{10}C_1[2^9 - 2] = 5100$

If group C has two students then number of groups

${}^{10}C_2[2^8 - 2] = 11430$

If group C has three students then number of groups

$= {}^{10}C_3 \times [2^7 - 2] = 15120$

So total groups = 31650

9. Let $i=\sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}=k$, and n = [| k |] be the greatest integral part of | k |. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to __________.

Official Ans. by NTA (310)

Sol. $$K=\frac{1}{2^9}\left[\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{21}}{\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)^{24}}+\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{21}}{\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)^{24}}\right]$$

$$K=\frac{1}{512}\left[\frac{\left(e^{i\frac{2\pi}{3}}\right)^{21}}{\left(e^{-\frac{i\pi}{4}}\right)^{24}}+\frac{\left(e^{\frac{i\pi}{3}}\right)^{21}}{\left(e^{\frac{i\pi}{4}}\right)^{24}}\right]$$

$$K=\frac{1}{512}\left[e^{i(14\pi+6\pi)}+e^{i(7\pi-6\pi)}\right]$$

$$K=\frac{1}{512}\left[e^{20\pi i}+e^{\pi i}\right]$$

$$K=\frac{1}{512}[1+(-1)]=0$$

n = [|k|] = 0

$$\sum_{j=0}^{5}(j+5)^2-\sum_{j=0}^{5}(j+5)$$

$$\sum_{j=0}^{5}(j^2+25+10j-j-5)$$

$$\sum_{j=0}^{5}(j^2+9j+20)$$

$$\sum_{j=0}^{5}j^2+9\sum_{j=0}^{5}j+20\sum_{j=0}^{5}1$$

$$\frac{5\times6\times11}{6}+9\left(\frac{5\times6}{2}\right)+20\times6$$

= 55 + 135 + 120

= 310

10. The number of the real roots of the equation $(x+1)^2+|x-5|=\frac{27}{4}$ is ________.

Official Ans. by NTA (2)

Sol. **Case-I**

$x \le 5$

$$(x+1)^2-(x-5)=\frac{27}{4}$$

$$(x+1)^2-(x+1)-\frac{3}{4}=0$$

$$x+1=\frac{3}{2},-\frac{1}{2}$$

$$x=\frac{1}{2},-\frac{3}{2}$$

Case-II

$x > 5$

$$(x+1)+(x-5)=\frac{27}{4}$$

$$(x+1)^2+(x+1)-\frac{51}{4}=0$$

$$x=\frac{-1\pm\sqrt{52}}{2}\ \text{(rejected as } x>5)$$

So, the equation have two real root.