

**FINAL JEE-MAIN EXAMINATION – FEBRUARY, 2021**

(Held On Thursday 25<sup>th</sup> February, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

**MATHEMATICS**

**TEST PAPER WITH SOLUTION**

**SECTION-A**

1. When a missile is fired from a ship, the probability that it is intercepted is  $\frac{1}{3}$  and the probability that the missile hits the target, given that it is not intercepted, is  $\frac{3}{4}$ . If three missiles are fired independently from the ship, then the probability that all three hit the target, is :

- (1)  $\frac{1}{27}$       (2)  $\frac{3}{4}$       (3)  $\frac{1}{8}$       (4)  $\frac{3}{8}$

**Official Ans. by NTA (3)**

**Sol.** Required probability =  $\left(\frac{2}{3} \times \frac{3}{4}\right)^3 = \frac{1}{8}$

2. If  $0 < \theta, \phi < \frac{\pi}{2}$ ,  $x = \sum_{n=0}^{\infty} \cos^{2n} \theta$ ,  $y = \sum_{n=0}^{\infty} \sin^{2n} \phi$

and  $z = \sum_{n=0}^{\infty} \cos^{2n} \theta \cdot \sin^{2n} \phi$  then :

- (1)  $xy - z = (x + y)z$       (2)  $xy + yz + zx = z$   
(3)  $xyz = 4$       (4)  $xy + z = (x + y)z$

**Official Ans. by NTA (4)**

**Sol.**  $x = \frac{1}{1 - \cos^2 \theta} \Rightarrow \sin^2 \theta = \frac{1}{x}$

Also,  $\cos^2 \theta = \frac{1}{y}$  &  $1 - \sin^2 \theta \cos^2 \theta = \frac{1}{z}$

So,  $1 - \frac{1}{x} \times \frac{1}{y} = \frac{1}{z} \Rightarrow z(xy - 1) = xy \dots(1)$

Also,  $\frac{1}{x} + \frac{1}{y} = 1 \Rightarrow x + y = xy \dots(2)$

From (i) and (ii)

$xy + z = xyz = (x + y)z$

3. Let  $f, g : \mathbb{N} \rightarrow \mathbb{N}$  such that  $f(n + 1) = f(n) + f(1)$   
 $\forall n \in \mathbb{N}$  and  $g$  be any arbitrary function.  
Which of the following statements is NOT true?

- (1) If  $f \circ g$  is one-one, then  $g$  is one-one  
(2) If  $f$  is onto, then  $f(n) = n \forall n \in \mathbb{N}$   
(3)  $f$  is one-one  
(4) If  $g$  is onto, then  $f \circ g$  is one-one

**Official Ans. by NTA (4)**

**Sol.**  $f(n + 1) - f(n) = f(1)$   
 $\Rightarrow f(n) = nf(1)$

$\Rightarrow f$  is one-one

Now, Let  $f(g(x_2)) = f(g(x_1))$

$\Rightarrow g(x_2) = g(x_1)$  (as  $f$  is one-one)

$\Rightarrow x_1 = x_2$  (as  $f \circ g$  is one-one)

$\Rightarrow g$  is one-one

Now,  $f(g(n)) = g(n) f(1)$

may be many-one if

$g(n)$  is many-one

4. The equation of the line through the point  $(0, 1, 2)$  and perpendicular to the line

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$  is :

(1)  $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

(2)  $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

(3)  $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

(4)  $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

**Official Ans. by NTA (4)**

**Sol.**  $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = r$

$\Rightarrow P(x, y, z) = (2r + 1, 3r - 1, -2r + 1)$

Since,  $\overline{QP} \perp (2\hat{i} + 3\hat{j} - 2\hat{k})$

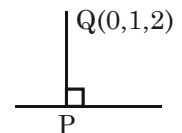
$\Rightarrow 4r + 2 + 9r - 6 + 4r + 2 = 0$

$\Rightarrow r = \frac{2}{17}$

$\Rightarrow P\left(\frac{21}{17}, \frac{-11}{17}, \frac{13}{17}\right)$

$\Rightarrow \overline{PQ} = \frac{21\hat{i} - 28\hat{j} - 21\hat{k}}{17}$

So,  $\overline{QP} : \frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$



5. Let  $\alpha$  be the angle between the lines whose direction cosines satisfy the equations  $l + m - n = 0$  and  $l^2 + m^2 - n^2 = 0$ . Then the

value of  $\sin^4\alpha + \cos^4\alpha$  is :

- (1)  $\frac{3}{4}$       (2)  $\frac{3}{8}$       (3)  $\frac{5}{8}$       (4)  $\frac{1}{2}$

**Official Ans. by NTA (3)**

**Sol.**  $n = \ell + m$

Now,  $\ell^2 + m^2 = n^2 = (\ell + m)^2$

$\Rightarrow 2\ell m = 0$

If  $\ell = 0 \Rightarrow m = n = \pm \frac{1}{\sqrt{2}}$

And, If  $m = 0 \Rightarrow n = \ell = \pm \frac{1}{\sqrt{2}}$

So, direction cosines of two lines are

$\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$  and  $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

Thus,  $\cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

**6.** The value of the integral

$$\int \frac{\sin\theta \cdot \sin 2\theta (\sin^6\theta + \sin^4\theta + \sin^2\theta) \sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$$

is :

(where c is a constant of integration)

(1)  $\frac{1}{18} [11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta]^{\frac{3}{2}} + c$

(2)  $\frac{1}{18} [9 - 2\cos^6\theta - 3\cos^4\theta - 6\cos^2\theta]^{\frac{3}{2}} + c$

(3)  $\frac{1}{18} [9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta]^{\frac{3}{2}} + c$

(4)  $\frac{1}{18} [11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta]^{\frac{3}{2}} + c$

**Official Ans. by NTA (4)**

**Sol.**  $I = \int \frac{\sin\theta \cdot \sin 2\theta (\sin^6\theta + \sin^4\theta + \sin^2\theta) \sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$

$\Rightarrow I = \int \frac{\sin\theta \cdot 2\sin\theta \cos\theta \cdot \sin^2\theta (\sin^4\theta + \sin^2\theta + 1) (2\sin^4\theta + 3\sin^2\theta + 6)^{1/2}}{2\sin^2\theta} d\theta$

$= \int \sin^2\theta \cdot \cos\theta (\sin^4\theta + \sin^2\theta + 1) (2\sin^4\theta + 3\sin^2\theta + 6)^{1/2} d\theta$

Let  $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

$\therefore I = \int t^2 (t^4 + t^2 + 1) (2t^4 + 3t^2 + 6)^{1/2} dt$

$= \int (t^5 + t^3 + t) t (2t^4 + 3t^2 + 6)^{1/2} dt$

$= \int (t^5 + t^3 + t) (t^2)^{1/2} (2t^4 + 3t^2 + 6)^{1/2} dt$

$= \int (t^5 + t^3 + t) (2t^6 + 3t^4 + 6t^2)^{1/2} dt$

Let  $2t^6 + 3t^4 + 6t^2 = u^2$

$\Rightarrow 12(t^5 + t^3 + t) dt = 2udu$

$\therefore I = \int (u^2)^{1/2} \cdot \frac{2udu}{12}$

$= \int \frac{u^2}{6} du = \frac{u^3}{18} + C$

$= \frac{(2t^6 + 3t^4 + 6t^2)^{3/2}}{18} + C$

when  $t = \sin\theta$

and  $t^2 = 1 - \cos^2\theta$  will give option (4)

**7.** The value of  $\int_{-1}^1 x^2 e^{[x^3]} dx$ , where [t] denotes the greatest integer  $\leq t$ , is :

- (1)  $\frac{e-1}{3e}$       (2)  $\frac{e+1}{3}$       (3)  $\frac{e+1}{3e}$       (4)  $\frac{1}{3e}$

**Official Ans. by NTA (3)**

**Sol.**  $I = \int_{-1}^1 x^2 e^{[x^3]} dx$

$= \int_{-1}^0 x^2 e^{[x^3]} dx + \int_0^1 x^2 e^{[x^3]} dx$

$= \int_{-1}^0 x^2 e^{-1} dx + \int_0^1 x^2 e^0 dx$

$= \frac{1}{e} \times \frac{x^3}{3} \Big|_{-1}^0 + \frac{x^3}{3} \Big|_0^1$

$= \frac{1}{e} \times \left( 0 - \left( \frac{-1}{3} \right) \right) + \frac{1}{3}$

$= \frac{1}{3e} + \frac{1}{3} = \frac{1+e}{3e}$

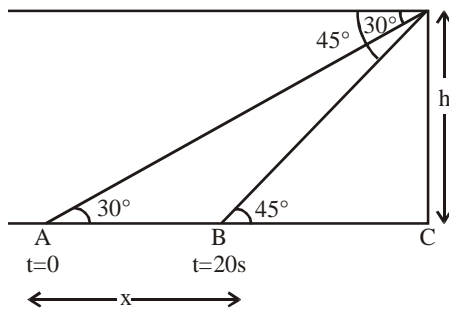
**8.** A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is

$30^\circ$  (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is  $45^\circ$ . Then the time taken (in seconds) by the boat from B to reach the base of the tower is:

- (1) 10                                      (2)  $10\sqrt{3}$   
 (3)  $10(\sqrt{3}+1)$                       (4)  $10(\sqrt{3}-1)$

**Official Ans. by NTA (3)**

**Sol.**



Let speed of boat is  $u$  m/s and height of tower is  $h$  meter & distance  $AB = x$  metre

$$\therefore x = h \cot 30^\circ - h \cot 45^\circ$$

$$\Rightarrow x = h(\sqrt{3}-1)$$

$$\therefore u = \frac{x}{20} = \frac{h(\sqrt{3}-1)}{20} \text{ m/s}$$

$\therefore$  Time taken to travel from B to C (Distance =  $h$  meter)

$$= \frac{h}{u} = \frac{h}{\frac{h(\sqrt{3}-1)}{20}} = \frac{20}{\sqrt{3}-1} = 10(\sqrt{3}+1) \text{ sec.}$$

9. A tangent is drawn to the parabola  $y^2 = 6x$  which is perpendicular to the line  $2x + y = 1$ . Which of the following points does NOT lie on it?

- (1)  $(-6, 0)$  (2)  $(4, 5)$  (3)  $(5, 4)$  (4)  $(0, 3)$

**Official Ans. by NTA (3)**

**Sol.** Slope of tangent =  $m_T = m$

$$\text{So, } m(-2) = -1 \Rightarrow m = \frac{1}{2}$$

$$\text{Equation : } y = mx + \frac{a}{m}$$

$$\Rightarrow y = \frac{1}{2}x + \frac{3}{2 \times \frac{1}{2}} \left( a = \frac{6}{4} = \frac{3}{2} \right)$$

$$\Rightarrow y = \frac{x}{2} + 3$$

$$\Rightarrow 2y = x + 6$$

Point  $(5, 4)$  will not lie on it

10. All possible values of  $\theta \in [0, 2\pi]$  for which  $\sin 2\theta + \tan 2\theta > 0$  lie in :

(1)  $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(2)  $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(3)  $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(4)  $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

**Official Ans. by NTA (4)**

**Sol.**  $\sin 2\theta + \tan 2\theta > 0$

$$\Rightarrow \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$$

$$\Rightarrow \sin 2\theta \frac{(\cos 2\theta + 1)}{\cos 2\theta} > 0 \Rightarrow \tan 2\theta (2 \cos^2 \theta) > 0$$

Note :  $\cos 2\theta \neq 0$

$$\Rightarrow 1 - 2 \sin^2 \theta \neq 0 \Rightarrow \sin \theta \neq \pm \frac{1}{\sqrt{2}}$$

Now,  $\tan 2\theta (1 + \cos 2\theta) > 0$

$$\Rightarrow \tan 2\theta > 0 \quad (\text{as } \cos 2\theta + 1 > 0)$$

$$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$$

As  $\sin\theta \neq \pm \frac{1}{\sqrt{2}}$ ; which has been already considered

11. Let the lines  $(2 - i)z = (2 + i)\bar{z}$  and  $(2 + i)z + (i - 2)\bar{z} - 4i = 0$ , (here  $i^2 = -1$ ) be normal to a circle C. If the line  $iz + \bar{z} + 1 + i = 0$  is tangent to this circle C, then its radius is:

- (1)  $\frac{3}{\sqrt{2}}$     (2)  $\frac{1}{2\sqrt{2}}$     (3)  $3\sqrt{2}$     (4)  $\frac{3}{2\sqrt{2}}$

**Official Ans. by NTA (4)**

- Sol. (i)  $(2 - i)z = (2 + i)\bar{z}$

$$\boxed{y = \frac{x}{2}}$$

- (ii)  $(2 + i)z + (i - 2)\bar{z} - 4i = 0$

$$\boxed{x + 2y = 2}$$

- (iii)  $iz + \bar{z} + 1 + i = 0$

Eq<sup>n</sup> of tangent  $\boxed{x - y + 1 = 0}$

Solving (i) and (ii)

$$x = 1, y = \frac{1}{2}$$

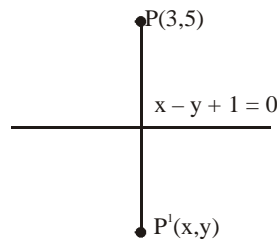
Now,  $p = r \Rightarrow \left| \frac{1 - \frac{1}{2} + 1}{\sqrt{2}} \right| = r$

$$\Rightarrow r = \frac{3}{2\sqrt{2}}$$

12. The image of the point (3, 5) in the line  $x - y + 1 = 0$ , lies on :

- (1)  $(x - 2)^2 + (y - 2)^2 = 12$   
 (2)  $(x - 4)^2 + (y + 2)^2 = 16$   
 (3)  $(x - 4)^2 + (y - 4)^2 = 8$   
 (4)  $(x - 2)^2 + (y - 4)^2 = 4$

**Official Ans. by NTA (4)**



Sol.

$$\frac{x-3}{1} = \frac{y-5}{-1} = -2 \left( \frac{3-5+1}{1+1} \right)$$

So,  $x = 4, y = 4$

Hence,  $(x - 2)^2 + (y - 4)^2 = 4$

13. If the curves,  $\frac{x^2}{a} + \frac{y^2}{b} = 1$  and  $\frac{x^2}{c} + \frac{y^2}{d} = 1$  intersect each other at an angle of  $90^\circ$ , then which of the following relations is TRUE?

- (1)  $a + b = c + d$   
 (2)  $a - b = c - d$   
 (3)  $a - c = b + d$   
 (4)  $ab = \frac{c+d}{a+b}$

**Official Ans. by NTA (2)**

- Sol. For orthogonal curves  $a - c = b - d$   
 $\Rightarrow a - b = c - d$

14.  $\lim_{n \rightarrow \infty} \left( 1 + \frac{1 + \frac{1}{2} + \dots + \frac{1}{n}}{n^2} \right)^n$  is equal to :

- (1)  $\frac{1}{2}$     (2) 0    (3)  $\frac{1}{e}$     (4) 1

**Official Ans. by NTA (4)**

- Sol. Given limit is of  $1^\infty$  form

So,  $l = \exp \left( \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}}{n} \right)$

Now,

$$0 \leq 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \leq 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} \leq 2\sqrt{n} - 1$$

So,  $l = \exp(0)$  (from sandwich theorem)  
 $= 1$

15. The coefficients a, b and c of the quadratic equation,  $ax^2 + bx + c = 0$  are obtained by throwing a dice three times. The probability that this equation has equal roots is:

- (1)  $\frac{1}{72}$       (2)  $\frac{5}{216}$       (3)  $\frac{1}{36}$       (4)  $\frac{1}{54}$

**Official Ans. by NTA (2)**

**Sol.**  $ax^2 + bx + c = 0$

For equal roots  $D = 0$

$$\Rightarrow b^2 = 4ac$$

**Case I :**  $ac = 1$

(a, b, c) = (1, 2, 1)

**Case II :**  $ac = 4$

(a, b, c) = (1, 4, 4)

or (4, 4, 1)

or (2, 4, 2)

**Case III :**  $ac = 9$

(a, b, c) = (3, 6, 3)

$$\text{Required probability} = \frac{5}{216}$$

16. The total number of positive integral solutions (x, y, z) such that  $xyz = 24$  is :

- (1) 36      (2) 24      (3) 45      (4) 30

**Official Ans. by NTA (4)**

**Sol.**  $xyz = 2^3 \times 3^1$

$$\text{Let } x = 2^{\alpha_1} \times 3^{\beta_1}$$

$$y = 2^{\alpha_2} \times 3^{\beta_2}$$

$$z = 2^{\alpha_3} \times 3^{\beta_3}$$

$$\text{Now } \alpha_1 + \alpha_2 + \alpha_3 = 3.$$

$$\text{No. of non-negative integral sol} = {}^5C_2 = 10$$

$$\& \beta_1 + \beta_2 + \beta_3 = 1$$

$$\text{No. of non-negative integral sol}^n = {}^3C_2 = 3$$

$$\text{Total ways} = 10 \times 3 = 30.$$

17. The integer 'k', for which the inequality  $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$  is valid for every x in R, is :

- (1) 3      (2) 2      (3) 0      (4) 4

**Official Ans. by NTA (1)**

**Sol.**  $x^2 - 2(3K - 1)x + 8K^2 - 7 > 0$

Now,  $D < 0$

$$\Rightarrow 4(3K - 1)^2 - 4 \times 1 \times (8K^2 - 7) < 0$$

$$\Rightarrow 9K^2 - 6K + 1 - 8K^2 + 7 < 0$$

$$\Rightarrow K^2 - 6K + 8 < 0$$

$$\Rightarrow (K - 4)(K - 2) < 0$$

$$\Rightarrow \boxed{K \in (2, 4)}$$

18. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is

$$\frac{x^2 - 4x + y + 8}{x - 2}, \text{ then this curve also passes}$$

through the point:

(1) (5, 4)      (2) (4, 5)

(3) (4, 4)      (4) (5, 5)

**Official Ans. by NTA (4)**

**Sol.** Given

$$y(0) = 0$$

$$\& \frac{dy}{dx} = \frac{(x-2)^2 + y + 4}{x-2}$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x-2} = (x-2) + \frac{4}{x-2}$$

$$\Rightarrow \text{I.F.} = e^{-\int \frac{1}{x-2} dx} = \frac{1}{x-2}$$

Solution of L.D.E.

$$\Rightarrow y \frac{1}{x-2} = \int \frac{1}{x-2} \left( (x-2) + \frac{4}{x-2} \right) dx$$

$$\Rightarrow \frac{y}{x-2} = x - \frac{4}{x-2} + C$$

$$\text{Now, at } x = 0, y = 0 \Rightarrow C = -2$$

$$y = x(x-2) - 4 - 2(x-2)$$

$$\Rightarrow y = x^2 - 4x$$

This curve passes through (5, 5)

19. The statement  $A \rightarrow (B \rightarrow A)$  is equivalent to :

(1)  $A \rightarrow (A \wedge B)$       (2)  $A \rightarrow (A \rightarrow B)$

(3)  $A \rightarrow (A \leftrightarrow B)$       (4)  $A \rightarrow (A \vee B)$

**Official Ans. by NTA (4)**

**Sol.**  $A \rightarrow (B \rightarrow A)$

$$\equiv A \rightarrow (\sim B \vee A)$$

$$\equiv \sim A \vee (\sim B \vee A)$$

$$\equiv (\sim A \vee A) \vee \sim B$$

$$\equiv T \vee \sim B \equiv T$$

$$\therefore T \vee B = T$$

$$\equiv (\sim A \vee A) \vee B$$

$$\equiv \sim A \vee (A \vee B)$$

$$\equiv A \rightarrow (A \vee B)$$

20. If Rolle's theorem holds for the function  $f(x) = x^3 - ax^2 + bx - 4$ ,  $x \in [1, 2]$  with

$f'\left(\frac{4}{3}\right) = 0$ , then ordered pair  $(a, b)$  is equal to :

- (1) (5, 8) (2) (-5, 8)
(3) (5, -8) (4) (-5, -8)

Official Ans. by NTA (1)

Sol. f(1) = f(2)
=> 1 - a + b - 4 = 8 - 4a + 2b - 4
=> 3a - b = 7 .....(1)

Also f'(4/3) = 0 (given)

=> (3x^2 - 2ax + b)|\_{x=4/3} = 0

=> 16/3 - 8a/3 + b = 0

=> 8a - 3b - 16 = 0 ....(2)

Solving (1) and (2)

a = 5, b = 8

SECTION-B

1. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has

extrema at x = -1 and x = 1. If lim\_{x->0} f(x)/x^3 = 1, then

5\*f(2) is equal to \_\_\_\_\_.

Official Ans. by NTA (144)

Sol. Let f(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f

as lim\_{x->0} f(x)/x^3 = 1 non-zero finite

So, d = e = f = 0
and f(x) = x^3(x^3 + ax^2 + bx + c)

Hence, lim\_{x->0} f(x)/x^3 = c = 1

Now, as f(x) = x^6 + ax^5 + bx^4 + x^3
and f'(x) = 0 at x = 1 and x = -1

i.e., f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2

f'(1) = 0

=> 6 + 5a + 4b + 3 = 0

=> 5a + 4b = -9

& f'(-1) = 0

=> -6 + 5a - 4b + 3 = 0

=> 5a - 4b = 3

Solving both we get,

a = -6/10 = -3/5; b = -3/2

∴ f(x) = x^6 - 3/5 x^5 - 3/2 x^4 + x^3

∴ 5f(2) = 5[64 - 3/5 \* 32 - 3/2 \* 16 + 8]
= 320 - 96 - 120 + 40
= 144

2. The number of points, at which the function f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|, x ∈ R is not differentiable, is \_\_\_\_\_.

Official Ans. by NTA (2)

Sol. f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|
= |2x + 1| - 3|x + 2| + |x + 2||x - 1|
= |2x + 1| + |x + 2|(|x - 1| - 3)

Critical points are x = -1/2, -2, -1

but x = -2 is making a zero.

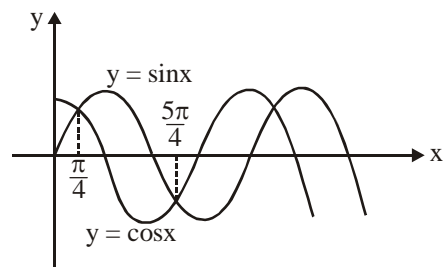
twice in product so, points of non

differentiability are x = -1/2 and x = -1

∴ Number of points of non-differentiability = 2

3. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to \_\_\_\_\_.

Official Ans. by NTA (64)



Sol.

A = integral from pi/4 to 5pi/4 of (sin x - cos x) dx

$$= (-\cos x - \sin x) \Big|_{\pi/4}^{5\pi/4}$$

$$= \left( -\left(\frac{-1}{\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{2}}\right) \right) - \left( -\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right) \right)$$

$$\Rightarrow A = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = (2\sqrt{2})^4 = 16 \times 4 = 64$$

4. Let  $A_1, A_2, A_3, \dots$  be squares such that for each  $n \geq 1$ , the length of the side of  $A_n$  equals the length of diagonal of  $A_{n+1}$ . If the length of  $A_1$  is 12 cm, then the smallest value of  $n$  for which area of  $A_n$  is less than one, is \_\_\_\_\_.

**Official Ans. by NTA (9)**

**Sol.** Let  $a_n$  be the side length of  $A_n$ .

$$\text{So, } a_n = \sqrt{2}a_{n+1}, a_1 = 12$$

$$\Rightarrow a_n = 12 \times \left(\frac{1}{\sqrt{2}}\right)^{n-1}$$

$$\text{Now, } (a_n)^2 < 1 \Rightarrow \frac{144}{2^{(n-1)}} < 1$$

$$\Rightarrow 2^{(n-1)} > 144$$

$$\Rightarrow n - 1 \geq 8$$

$$\Rightarrow n \geq 9$$

5. Let  $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ , where  $x, y$  and  $z$  are real

numbers such that  $x + y + z > 0$  and  $xyz = 2$ . If  $A^2 = I_3$ , then the value of  $x^3 + y^3 + z^3$  is \_\_\_\_\_.

**Official Ans. by NTA (7)**

**Sol.**  $A^2 = I$

$$\Rightarrow AA' = I \quad (\text{as } A' = A)$$

$\Rightarrow A$  is orthogonal

$$\text{So, } x^2 + y^2 + z^2 = 1 \text{ and } xy + yz + zx = 0$$

$$\Rightarrow (x + y + z)^2 = 1 + 2 \times 0$$

$$\Rightarrow x + y + z = 1$$

Thus,

$$x^3 + y^3 + z^3 = 3 \times 2 + 1 \times (1 - 0) = 7$$

6. If  $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$  and

$$(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}, \text{ then } 13(a^2 + b^2)$$

is equal to \_\_\_\_\_.

**Official Ans. by NTA (13)**

**Sol.**  $a^2 + b^2 = |I_2 + A| |I_2 - A|^{-1}$

$$= \sec^2 \frac{\theta}{2} \times \cos^2 \frac{\theta}{2} = 1$$

7. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is \_\_\_\_\_.

**Official Ans. by NTA (32)**

**Sol.** We need three digits numbers.

$$\text{Since } 1 + 2 + 3 + 4 + 5 = 15$$

So, number of possible triplets for multiple of 15 is  $1 \times 2 \times 2$

$$\text{so Ans.} = 4 \times \underline{3} + 4 \times 3 - 1 \times 2 \times \underline{2} = 32$$

8. Let  $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ ,  $\vec{b} = \hat{i} - \hat{j}$  and  $\vec{c} = \hat{i} - \hat{j} - \hat{k}$  be three given vectors. If  $\vec{r}$  is a vector such that  $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$  and  $\vec{r} \cdot \vec{b} = 0$ , then  $\vec{r} \cdot \vec{a}$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (12)**

**Sol.**  $(\vec{r} - \vec{c}) \times \vec{a} = 0$

$$\Rightarrow \vec{r} = \vec{c} + \lambda \vec{a}$$

$$\text{Now, } 0 = \vec{b} \cdot \vec{c} + \lambda \vec{a} \cdot \vec{b}$$

$$\Rightarrow \lambda = \frac{-\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} = \frac{-2}{-1} = 2$$

$$\text{So, } \vec{r} \cdot \vec{a} = \vec{a} \cdot \vec{c} + 2a^2 = 12$$

9. If the system of equations

$$kx + y + 2z = 1$$

$$3x - y - 2z = 2$$

$$-2x - 2y - 4z = 3$$

has infinitely many solutions, then  $k$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (21)**

**Sol.** We observe  $5P_2 - P_1 = 3P_3$

$$\text{So, } 15 - K = -6$$

$$\Rightarrow K = 21$$

10. The locus of the point of intersection of the lines

$$(\sqrt{3})kx + ky - 4\sqrt{3} = 0 \text{ and}$$

$\sqrt{3}x - y - 4(\sqrt{3})k = 0$  is a conic, whose eccentricity is \_\_\_\_\_.

**Official Ans. by NTA (2)**

**Sol.** 
$$K = \frac{4\sqrt{3}}{\sqrt{3}x + y} = \frac{\sqrt{3}x - y}{4\sqrt{3}}$$

$$\Rightarrow 3x^2 - y^2 = 48$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$$

$$\text{Now, } 48 = 16(e^2 - 1)$$

$$\Rightarrow e = \sqrt{4} = 2$$