

FINAL JEE-MAIN EXAMINATION – FEBRUARY, 2021

(Held On Friday 26th February, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

SECTION-A

1. If \vec{a} and \vec{b} are perpendicular, then $\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})))$ is equal to

- (1) $\vec{0}$ (2) $\frac{1}{2}|\vec{a}|^4 \vec{b}$ (3) $\vec{a} \times \vec{b}$ (4) $|\vec{a}|^4 \vec{b}$

Official Ans. by NTA (4)

Sol. $\vec{a} \cdot \vec{b} = 0$

$$\vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = -|\vec{a}|^2 \vec{b}$$

$$\text{Now } \vec{a} \times (\vec{a} \times (-|\vec{a}|^2 \vec{b}))$$

$$= -|\vec{a}|^2 (\vec{a} \times (\vec{a} \times \vec{b}))$$

$$= -|\vec{a}|^2 (-|\vec{a}|^2 \vec{b}) = |\vec{a}|^4 \vec{b}$$

2. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is

- (1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$ (3) $\frac{15}{2^8}$ (4) $\frac{15}{2^{14}}$

Official Ans. by NTA (1)

Sol. Let the coin be tossed n-times

$$P(H) = P(T) = \frac{1}{2}$$

$$P(7 \text{ heads}) = {}^n C_7 \left(\frac{1}{2}\right)^{n-7} \left(\frac{1}{2}\right)^7 = \frac{{}^n C_7}{2^n}$$

$$P(9 \text{ heads}) = {}^n C_9 \left(\frac{1}{2}\right)^{n-9} \left(\frac{1}{2}\right)^9 = \frac{{}^n C_9}{2^n}$$

$$P(7 \text{ heads}) = P(9 \text{ heads})$$

$${}^n C_7 = {}^n C_9 \Rightarrow n = 16$$

$$P(2 \text{ heads}) = {}^{16} C_2 \left(\frac{1}{2}\right)^{14} \left(\frac{1}{2}\right)^2 = \frac{15 \times 8}{2^{16}}$$

$$P(2 \text{ heads}) = \frac{15}{2^{13}}$$

TEST PAPER WITH SOLUTION

3. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is
(1) 4 (2) 1 (3) 6 (4) 12

Official Ans. by NTA (1)

Sol. $A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}, \quad a, b, c \in I$

$$A^2 = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} a^2 + b^2 & b(a+c) \\ b(a+c) & b^2 + c^2 \end{pmatrix}$$

Sum of the diagonal entries of

$$A^2 = a^2 + 2b^2 + c^2$$

Given $a^2 + 2b^2 + c^2 = 1, a, b, c \in I$

$$b = 0 \text{ \& } a^2 + c^2 = 1$$

Case-1 : $a = 0 \Rightarrow c = \pm 1$ (2-matrices)

Case-2 : $c = 0 \Rightarrow a = \pm 1$ (2-matrices)

Total = 4 matrices

4. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product

of the third and fifth term is 25. Then, the sum of 4th, 6th and 8th terms is equal to

- (1) 30 (2) 26 (3) 35 (4) 32

Official Ans. by NTA (3)

Sol. a, ar, ar^2, \dots

$$T_2 + T_6 = \frac{25}{2} \Rightarrow ar(1+r^4) = \frac{25}{2}$$

$$a^2 r^2 (1+r^4)^2 = \frac{625}{4} \quad \dots (1)$$

$$T_3 \cdot T_5 = 25 \Rightarrow (ar^2)(ar^4) = 25$$

$$a^2 r^6 = 25 \quad \dots (2)$$

On dividing (1) by (2)

$$\frac{(1+r^4)^2}{r^4} = \frac{25}{4}$$

$$4r^8 - 17r^4 + 4 = 0$$

$$(4r^4 - 1)(r^4 - 4) = 0$$

$$r^4 = \frac{1}{4}, 4 \Rightarrow r^4 = 4$$

(an increasing geometric series)

$$a^2 r^6 = 25 \Rightarrow (ar^3)^2 = 25$$

$$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7$$

$$= ar^3(1 + r^2 + r^4)$$

$$= 5(1 + 2 + 4) = 35$$

5. The value of $\sum_{n=1}^{100} \int_{n-1}^n e^{x-[x]} dx$, where $[x]$ is the

greatest integer $\leq x$, is

- (1) $100(e - 1)$ (2) $100(1 - e)$
 (3) $100e$ (4) $100(1 + e)$

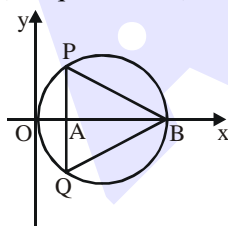
Official Ans. by NTA (1)

Sol. $\sum_{n=1}^{100} \int_{n-1}^n e^{(x)} dx$, period of $\{x\} = 1$

$$\sum_{n=1}^{100} \int_0^1 e^{(x)} dx = \sum_{n=1}^{100} \int_0^1 e^x dx$$

$$\sum_{n=1}^{100} (e - 1) = 100(e - 1)$$

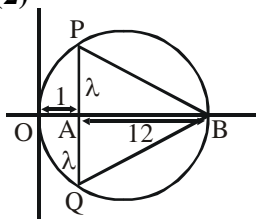
6. In the circle given below, let $OA = 1$ unit, $OB = 13$ unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is



- (1) $24\sqrt{2}$ (2) $24\sqrt{3}$
 (3) $26\sqrt{3}$ (4) $26\sqrt{2}$

Official Ans. by NTA (2)

Sol. $PA = AQ = \lambda$
 $OA \cdot AB$
 $= AP \cdot AQ$
 $\Rightarrow 1 \cdot 12 = \lambda \cdot \lambda$
 $\Rightarrow \lambda = 2\sqrt{3}$



$$\text{Area } \Delta PQB = \frac{1}{2} \times 2\lambda \times AB$$

$$\Delta = \frac{1}{2} \cdot 4\sqrt{3} \times 12$$

$$= 24\sqrt{3}$$

7. The sum of the infinite series

$$1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \dots$$
 is equal to

- (1) $\frac{13}{4}$ (2) $\frac{9}{4}$ (3) $\frac{15}{4}$ (4) $\frac{11}{4}$

Official Ans. by NTA (1)

Sol. $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \dots$

$$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \dots$$

$$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \frac{5}{3^4} + \dots + \text{up to infinite terms}$$

$$\Rightarrow S = \frac{13}{4}$$

8. The value of

$$\lim_{h \rightarrow 0} 2 \left\{ \frac{\sqrt{3} \sin\left(\frac{\pi}{6} + h\right) - \cos\left(\frac{\pi}{6} + h\right)}{\sqrt{3}h(\sqrt{3} \cosh - \sinh)} \right\}$$
 is

- (1) $\frac{4}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{3}{4}$ (4) $\frac{2}{3}$

Official Ans. by NTA (1)

Sol. $L = \lim_{h \rightarrow 0} 2 \left[\frac{\sqrt{3} \left(\frac{1}{2} \cosh + \frac{\sqrt{3}}{2} \sinh \right) - \left(\frac{\sqrt{3}}{2} \cosh - \frac{\sinh}{2} \right)}{(\sqrt{3}h)(\sqrt{3})} \right]$

$$L = \lim_{h \rightarrow 0} \frac{4 \sinh}{3h}$$

$$\Rightarrow L = \frac{4}{3}$$

9. The maximum value of the term independent of

't' in the expansion of $\left(tx^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t} \right)^{10}$

where $x \in (0,1)$ is

- (1) $\frac{10!}{\sqrt{3}(5!)^2}$ (2) $\frac{2 \cdot 10!}{3\sqrt{3}(5!)^2}$
 (3) $\frac{2 \cdot 10!}{3(5!)^2}$ (4) $\frac{10!}{3(5!)^2}$

Official Ans. by NTA (2)

Sol. Term independent of t will be the middle term due to exact same magnitude but opposite sign powers of t in the binomial expression given

so $T_6 = {}^{10}C_5 (tx^{\frac{1}{5}})^5 \left(\frac{(1-x)^{\frac{1}{10}}}{t} \right)^5$

$T_6 = f(x) = {}^{10}C_5 (x\sqrt{1-x})$; for maximum

$f'(x) = 0 \Rightarrow x = \frac{2}{3}$ & $f''\left(\frac{2}{3}\right) < 0$

so $f(x)_{\max} = {}^{10}C_5 \left(\frac{2}{3}\right) \cdot \frac{1}{\sqrt{3}}$

10. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after

$\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

- (1) 4 (2) 8 (3) 2 (4) 16

Official Ans. by NTA (1)

Sol. $\frac{dB}{dt} = \lambda B \Rightarrow \int_{1000}^{1200} \frac{dB}{B} = \lambda \int_0^2 dt \Rightarrow \lambda = \frac{1}{2} \ln\left(\frac{6}{5}\right)$

$\int_{1000}^{2000} \frac{dB}{B} = \frac{1}{2} \ln\left(\frac{6}{5}\right) \int_0^T dt \Rightarrow T = \frac{2 \ln 2}{\ln\left(\frac{6}{5}\right)}$

$\Rightarrow k = 2 \ln 2$

11. If $(1,5,35)$, $(7,5,5)$, $(1,\lambda,7)$ and $(2\lambda,1,2)$ are coplanar, then the sum of all possible values of λ is

- (1) $\frac{39}{5}$ (2) $-\frac{39}{5}$ (3) $\frac{44}{5}$ (4) $-\frac{44}{5}$

Official Ans. by NTA (3)

Sol. $A(1, 5, 35)$, $B(7, 5, 5)$, $C(1, \lambda, 7)$, $D(2\lambda, 1, 2)$

$\overline{AB} = 6\hat{i} - 30\hat{k}$, $\overline{BC} = -6\hat{i}(\lambda - 5)\hat{j} + 2\hat{k}$,

$\overline{CD} = (2\lambda - 1)\hat{i} + (1 - \lambda)\hat{j} - 5\hat{k}$

Points are coplanar

$\Rightarrow 0 = \begin{vmatrix} 6 & 0 & -30 \\ -6 & \lambda - 5 & 2 \\ 2\lambda - 1 & 1 - \lambda & -5 \end{vmatrix}$

$= 6(-5\lambda + 25 - 2 + 2\lambda) - 30(-6 + 6\lambda - (2\lambda^2 - \lambda - 10\lambda + 5))$
 $= 6(-3\lambda + 23) - 30(-2\lambda^2 + 11\lambda - 5 - 6 + 6\lambda)$
 $= 6(-3\lambda + 23) - 30(-2\lambda^2 + 17\lambda - 11)$
 $= 6(-3\lambda + 23 + 10\lambda^2 - 85\lambda + 55)$
 $= 6(10\lambda^2 - 88\lambda + 78) = 12(5\lambda^2 - 44\lambda + 39)$
 $\Rightarrow 0 = 12(5\lambda^2 - 44\lambda + 39)$

$\lambda_1 + \lambda_2 = \frac{44}{5}$

12. If $\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}$; $0 < x < 1$, then

the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is

(1) $\frac{1-y^2}{y\sqrt{y}}$ (2) $1 - y^2$

(3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{1-y^2}{2y}$

Official Ans. by NTA (3)

Sol. $\frac{\sin^{-1} x}{r} = a$, $\frac{\cos^{-1} x}{r} = b$, $\frac{\tan^{-1} y}{r} = c$

So, $a + b = \frac{\pi}{2r}$

$$\cos\left(\frac{\pi c}{a+b}\right) = \cos\left(\frac{\pi \tan^{-1} y}{\frac{\pi}{2r}}\right)$$

$$= \cos(2\tan^{-1}y), \text{ let } \tan^{-1}y = \theta$$

$$= \cos(2\theta)$$

$$= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1 - y^2}{1 + y^2}$$

13. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is

- (1) 42 (2) 82 (3) 77 (4) 35

Official Ans. by NTA (3)

- Sol. (I)** First possibility is 1, 1, 1, 1, 1, 2, 3

$$\text{required number} = \frac{7!}{5!} = 7 \times 6 = 42$$

- (II)** Second possibility is 1, 1, 1, 1, 2, 2, 2

$$\text{required number} = \frac{7!}{4! 3!} = \frac{7 \times 6 \times 5}{6} = 35$$

$$\text{Total} = 42 + 35 = 77$$

14. Let f be any function defined on \mathbb{R} and let it satisfy the condition :

$$|f(x) - f(y)| \leq |x - y|^2, \forall (x, y) \in \mathbb{R}$$

If $f(0) = 1$, then :

- (1) $f(x)$ can take any value in \mathbb{R}
 (2) $f(x) < 0, \forall x \in \mathbb{R}$
 (3) $f(x) = 0, \forall x \in \mathbb{R}$
 (4) $f(x) > 0, \forall x \in \mathbb{R}$

Official Ans. by NTA (4)

Sol. $\left| \frac{f(x) - f(y)}{(x - y)} \right| \leq |x - y|$

$$x - y = h \text{ let } \Rightarrow x = y + h$$

$$\lim_{x \rightarrow 0} \left| \frac{f(y+h) - f(y)}{h} \right| \leq 0$$

$$\Rightarrow |f'(y)| \leq 0 \Rightarrow f'(y) = 0$$

$$\Rightarrow f(y) = k \text{ (constant)}$$

and $f(0) = 1$ given

$$\text{So, } f(y) = 1 \Rightarrow f(x) = 1$$

15. The maximum slope of the curve

$$y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x \text{ occurs at the point}$$

- (1) (2,2) (2) (0,0)

- (3) (2,9) (4) $\left(3, \frac{21}{2}\right)$

Official Ans. by NTA (1)

Sol. $\frac{dy}{dx} = 2x^3 - 15x^2 + 36x - 19$

Since, slope is maximum so,

$$\frac{d^2y}{dx^2} = 6x^2 - 30x + 36 = 0$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$x = 2, 3$$

at $x = 2$

$$y = \frac{1}{2} \times 16 - 5 \times 8 + 18 \times 4 - 19 \times 2$$

$$= 8 - 40 + 72 - 38 = 80 - 78 = 2$$

point (2, 2)

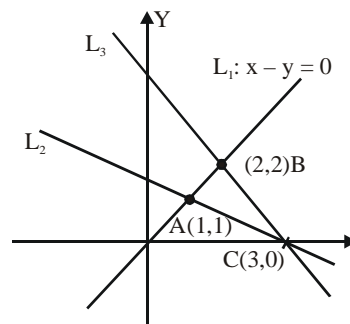
16. The intersection of three lines

$$x - y = 0, x + 2y = 3 \text{ and } 2x + y = 6 \text{ is a}$$

- (1) Right angled triangle
 (2) Equilateral triangle
 (3) Isosceles triangle
 (4) None of the above

Official Ans. by NTA (3)

Sol.



$$L_1 : x - y = 0$$

$$L_2 : x + 2y = 3$$

$$L_3 : x + y = 6$$

on solving L_1 and L_2 :

$$y = L \text{ and } x = 1$$

L_1 and L_3 :

$$x = 2$$

$$y = 2$$

L_2 and L_3 :

$$x + y = 3$$

$$2x + y = 6$$

$$x = 3$$

$$y = 0$$

$$AC = \sqrt{4+1} = \sqrt{5}$$

$$BC = \sqrt{4+1} = \sqrt{5}$$

$$AB = \sqrt{1+1} = \sqrt{2}$$

so its an isosceles triangle

17. Consider the three planes

$$P_1 : 3x + 15y + 21z = 9,$$

$$P_2 : x - 3y - z = 5, \text{ and}$$

$$P_3 : 2x + 10y + 14z = 5$$

Then, which one of the following is true ?

(1) P_1 and P_2 are parallel

(2) P_1 and P_3 are parallel

(3) P_2 and P_3 are parallel

(4) P_1, P_2 and P_3 all are parallel

Official Ans. by NTA (2)

Sol. $P_1 : x + 5y + 7z = 3,$

$$P_2 : x - 3y - z = 5$$

$$P_3 : x + 5y + 7z = \frac{5}{2}$$

so P_1 and P_3 are parallel.

18. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

(1) $(a+2)(a+3)(a+4)$

(2) -2

(3) $(a+1)(a+2)(a+3)$

(4) 0

Official Ans. by NTA (2)

Sol. $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$$\Delta = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3-a-1) & 1 & 0 \\ a^2+7a+12-a^2-3a-2 & 2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix}$$

$$= 4(a+2) - 4a - 10$$

$$= 4a + 8 - 4a - 10 = -2$$

19. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ is

- (1) $\frac{\pi}{4}$ (2) 4π (3) $\frac{\pi}{2}$ (4) 2π

Official Ans. by NTA (1)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ (using king)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^{-x}} dx = \int_{-\pi/2}^{\pi/2} \frac{3^x \cos^2 x}{1+3^x} dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \frac{(1+3^x) \cos^2 x}{1+3^x} dx$$

$$= \int_{-\pi/2}^{\pi/2} \cos^2 x dx = 2 \int_0^{\pi/2} \cos^2 x dx$$

$$\Rightarrow I = \int_0^{\pi/2} \cos^2 x dx = \frac{\pi}{4}$$

20. Let $R = \{(P,Q) \mid P \text{ and } Q \text{ are at the same distance from the origin}\}$ be a relation, then the equivalence class of $(1,-1)$ is the set :

(1) $S = \{(x,y) \mid x^2 + y^2 = 4\}$

(2) $S = \{(x,y) \mid x^2 + y^2 = 1\}$

(3) $S = \{(x,y) \mid x^2 + y^2 = \sqrt{2}\}$

(4) $S = \{(x,y) \mid x^2 + y^2 = 2\}$

Official Ans. by NTA (4)

Sol. Equivalence class of $(1, -1)$ is a circle with centre at $(0,0)$ and radius $= \sqrt{2}$
 $\Rightarrow x^2 + y^2 = 2$
 $S = \{(x,y) | x^2 + y^2 = 2\}$

SECTION-B

1. The difference between degree and order of a differential equation that represents the family of

curves given by $y^2 = a \left(x + \frac{\sqrt{a}}{2} \right), a > 0$ is

Official Ans. by NTA (2)

Sol. $y^2 = a \left(x + \frac{\sqrt{a}}{2} \right) = ax + \frac{a^{3/2}}{2} \dots(1)$

$\Rightarrow 2yy' = a$

put in equation (1)

$y^2 = (2yy')x + \frac{(2yy')^{3/2}}{2}$

$(y^2 - 2xyy') = \frac{(2yy')^{3/2}}{2}$

squaring

$(y^2 - 2xyy')^2 = \frac{y^3 (y')^3}{2}$

\therefore order = 1

degree = 3

Degree - order = 3 - 1 = 2

2. The number of integral values of 'k' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in \mathbb{R}$ is

Official Ans. by NTA (11)

Sol. $3\sin x + 4\cos x = k + 1$

$\Rightarrow k + 1 \in \left[-\sqrt{3^2 + 4^2}, \sqrt{3^2 + 4^2} \right]$

$\Rightarrow k + 1 \in [-5, 5]$

$\Rightarrow k \in [-6, 4]$

No. of integral values of $k = 11$

3. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is

Official Ans. by NTA (1)

Sol. $\log_4(x - 1) = \log_2(x - 3)$

$\Rightarrow \frac{1}{2} \log_2(x - 1) = \log_2(x - 3)$

$\Rightarrow \log_2(x - 1)^{1/2} = \log_2(x - 3)$

$\Rightarrow (x - 1)^{1/2} = x - 3$

$\Rightarrow x - 1 = x^2 + 9 - 6x$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x - 2)(x - 5) = 0$

$\Rightarrow x = 2, 5$

But $x \neq 2$ because it is not satisfying the domain of given equation i.e $\log_2(x - 3) \rightarrow$ its domain $x > 3$

finally x is 5

\therefore No. of solutions = 1.

4. The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is

Official Ans. by NTA (3)

Sol. $x^3 - 2x^2 + 2x - 1 = 0$

$x = 1$ satisfying the equation

$\therefore x - 1$ is factor of

$x^3 - 2x^2 + 2x - 1$

$= (x - 1)(x^2 - x + 1) = 0$

$x = 1, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}$

$x = 1, -\omega^2, -\omega$

sum of 162th power of roots

$= (1)^{162} + (-\omega^2)^{162} + (-\omega)^{162}$

$= 1 + (\omega)^{324} + (\omega)^{162}$

$= 1 + 1 + 1 = 3$

5. Let $m, n \in \mathbb{N}$ and $\gcd(2, n) = 1$. If

$$30 \binom{30}{0} + 29 \binom{30}{1} + \dots + 2 \binom{30}{28} + 1 \binom{30}{29} = n \cdot 2^m,$$

then $n + m$ is equal to

$$\left(\text{Here } \binom{n}{k} = {}^n C_k \right)$$

Official Ans. by NTA (45)

Sol. $30 \binom{30}{0} + 29 \binom{30}{1} + \dots + 2 \binom{30}{28} + 1 \binom{30}{29}$
 $= 30 \binom{30}{30} + 29 \binom{30}{29} + \dots + 2 \binom{30}{2} + 1 \binom{30}{1}$

$$= \sum_{r=1}^{30} r \binom{30}{r}$$

$$= \sum_{r=1}^{30} r \binom{30}{r} \binom{29}{r-1}$$

$$= 30 \sum_{r=1}^{30} \binom{29}{r-1}$$

$$= 30 \left(\binom{29}{0} + \binom{29}{1} + \binom{29}{2} + \dots + \binom{29}{29} \right)$$

$$= 30 \left(2^{29} \right) = 15 \left(2 \right)^{30} = n \left(2 \right)^m$$

$$\therefore n = 15, m = 30$$

$$n + m = 45$$

6. If $y = y(x)$ is the solution of the equation

$$e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x, y(0) = 0;$$

$$\text{then } 1 + y \left(\frac{\pi}{6} \right) + \frac{\sqrt{3}}{2} y \left(\frac{\pi}{3} \right) + \frac{1}{\sqrt{2}} y \left(\frac{\pi}{4} \right) \text{ is}$$

equal to

Official Ans. by NTA (1)

Sol. Put $e^{\sin y} = t$

$$\Rightarrow e^{\sin y} \cos y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \text{D.E is } \frac{dt}{dx} + t \cos x = \cos x$$

$$\text{I.F.} = e^{\int \cos x dx} = e^{\sin x}$$

$$\Rightarrow \text{solution is } t \cdot e^{\sin x} = \int \cos x e^{\sin x}$$

$$\Rightarrow e^{\sin y} e^{\sin x} = e^{\sin x} + c$$

$$\because x = 0, y = 0 \Rightarrow c = 0$$

$$\Rightarrow e^{\sin y} = 1$$

$$\Rightarrow y = 0$$

$$\Rightarrow 1 + y \left(\frac{\pi}{6} \right) + \frac{\sqrt{3}}{2} y \left(\frac{\pi}{3} \right) + \frac{1}{\sqrt{2}} y \left(\frac{\pi}{4} \right) = 1$$

7. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the points $(-2, -21, 29)$ and $(-1, -16, 23)$, then

$$\left(\frac{\lambda}{11} \right)^2 - \frac{4\lambda}{11} - 4 \text{ is equal to}$$

Official Ans. by NTA (8)

$$\begin{cases} A(-2, -21, 29) \\ B(-1, -16, 23) \end{cases}$$

Sol.

$$\begin{matrix} P(\lambda, 2, 1) & Q(4, -2, 2) \end{matrix}$$

$$\overline{AB} \cdot \overline{PQ} = 0$$

$$\Rightarrow (\hat{i} + 5\hat{j} - 6\hat{k}) \cdot ((4 - \lambda)\hat{i} - 4\hat{j} + \hat{k}) = 0$$

$$\Rightarrow 4 - \lambda - 20 - 6 = 0$$

$$\Rightarrow \lambda = -22$$

$$\Rightarrow \left(\frac{\lambda}{11} \right)^2 - \frac{4\lambda}{11} - 4 = 4 + 8 - 4 = 8$$

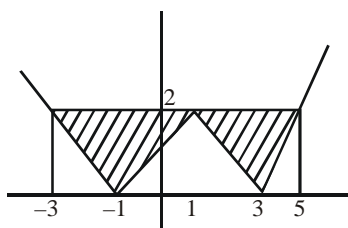
8. The area bounded by the lines $y = |x - 1| - 2$ is

Official Ans. by NTA (8)

Ans. By ALLEN (BONUS)

Sol. Remark :

Question is incomplete it should be area bounded by $y = |x - 1| - 2$ and $y = 2$



$$\text{Area} = 2 \left(\frac{1}{2} \cdot 4 \cdot 2 \right)$$

9. The value of the integral $\int_0^{\pi} |\sin 2x| dx$ is

Official Ans. by NTA (2)

Sol. Put $2x = t \Rightarrow 2dx = dt$

$$\Rightarrow I = \frac{1}{2} \int_0^{2\pi} |\sin t| dt$$

$$= \int_0^{\pi} |\sin t| dt$$

$$= 2$$

10. If $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$, the number of solutions of the given equation when

$$x \in \left[0, \frac{\pi}{2} \right] \text{ is}$$

Official Ans. by NTA (1)

$$\text{Sol. } \sqrt{3}(\cos x)^2 - \sqrt{3}\cos x + \cos x - 1 = 0$$

$$\Rightarrow (\sqrt{3}\cos x + 1)(\cos x - 1) = 0$$

$$\Rightarrow \cos x = 1 \text{ or } \cos x = -\frac{1}{\sqrt{3}} \text{ (reject)}$$

$$\Rightarrow x = 0 \text{ only}$$