10. (a) Show that the equation $yz + zx + xy = 0$ represents a right circular cone. Find its axis and semi-vertical angle. (10)

(b) Find the equation of a right circular cone which passes through (2, 1, 3) and has its vertex at the point (1, 1, 2) and axis the line

$$\frac{x-1}{2} = \frac{y-1}{-4} = \frac{z-2}{3}$$ (10)

Register Number :

Name of the Candidate :

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B.Sc. DEGREE EXAMINATION, 2012

(MATHEMATICS)

(SECOND YEAR)

(PART - III)

(PAPER - III)

650. ALGEBRA AND SOLID GEOMETRY

(Including Lateral Entry)

May] [Time : 3 Hours

Maximum : 100 Marks

Answer any FIVE questions.
ALL questions carry equal marks.

($5 \times 20 = 100$)

1. (a) Solve the equation

$$x^5 - x^4 + 8x^2 - 9x - 15 = 0,$$

if $-\sqrt{3}$ and $1 + 2i$ are two of its roots. (7)

(b) If α, β, γ are the roots of

$x^3 + qx + r = 0$, find the value of

$(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$. (6)

(c) If the roots of the equation

$ax^3 + 3bx^2 + 3cx + d = 0$

are in geometric progression,

prove that $\left(\frac{a}{d}\right) = \left(\frac{b}{c}\right)^3$ (7).

2. (a) Solve $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$

(10)

(b) If α, β, γ are the roots of the equation

$$x^3 + px^2 + qx + r = 0,$$

find the equation whose roots are

$$\beta\gamma - \alpha^2,\ \gamma\alpha - \beta^2,\ \alpha\beta - \gamma^2. \quad (10)$$

3. (a) State and prove Fermat's theorem. (12)

(b) Show that $28! \equiv 666 \pmod{899}$. (8)

8. (a) Find, in symmetrical form, the equations of the orthogonal projection of the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{4}$$

on the plane $3x + 4y + 5z = 0$ (10)

(b) Find the shortest distance and the equation of the line of shortest distance between the straight line

$$\frac{x+3}{-4} = \frac{y-6}{6} = \frac{z}{2} \text{ and } \frac{x-2}{-4} = \frac{y}{1} = \frac{z-7}{1}$$

(10).

9. (a) A plane passes through a fixed piont (a, b, c) and cuts the axes in A, B, C. Show that locus of the centre of the sphere OABC is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$. (10)

(b) A sphere touches the plane

$x - 2y - 2z - 7 = 0$ at $(3, -1, -1)$ and passes through $(1, 1, -3)$. Find its equation.

(10)

(b) Show that a finite commutative ring R without zero-divisors is a field. (5)

(c) Let F be a field, let f(x) and g(x) be two polynomials in f[x] with $g(x) \neq 0$. Prove that there exist unique polynomials q(x) and r(x) such that $f(x) = q(x)\, g(x) + r(x)$ where either $r(x) = 0$ or $\deg r(x) < \deg g(x)$. (10)

7. (a) A line makes angles α, β, γ, δ with the four diagonals of a cube. Prove that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta =$$

$\frac{\odot}{3}$

(10)

(b) Find the image of the point (1, –2, 3) in the plane $2x - 3y + 2z + 3 = 0$. (5)

(c) Find the equation of the plane passing through the points (3, 1, 2), (3, 4, 4) and perpendicular to the plane

$$5x + y + 4z = 0. \quad (5)$$

4. (a) Show that a function $f: A \rightarrow B$ is a bijection if and only if there exists a unique function $g: B \rightarrow A$ such that, *gof* and *fog* are identity functions on A and B, respectively. (8)

(b) Prove that an equivalence relation R in a non-empty set S determines a partition of S and conversely, a partition of S defines an equivalence relations. (12)

5. (a) If H is a subgroup of a group G, show that the number of left cosets in H is the same as the number of right cosets of H. (7)

(b) Show that if a group G has exactly one subgroup H of given order, then H is a normal subgroup of G. (6)

(c) Prove that a non-empty subset H of a subgroup G is a subgroup of G if and only if

$$a, b \in H \Rightarrow ab^{-1} \in H \quad (7)$$

6. (a) Show that the ring (Zn, $\oplus$,   ) is an integral domain if and only if *n* is a prime number. (5)

Turn Over